Bollobás–Meir TSP conjecture holds asymptotically

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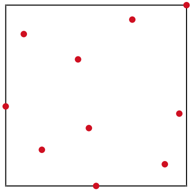
July 7, 2026

A worst-case TSP

? best $c(n)$: $\forall\ X \subseteq [0,1]^2$, $|X| = n$, $\exists$ Ham. cycle $\mathbf{H}$: $\sum_{\mathbf{H}} |e| \leq c(n)$



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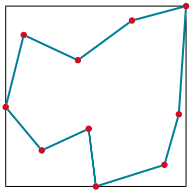

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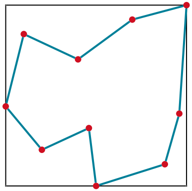

- $c(n) \geq 1.075\sqrt{n} - o(\sqrt{n})$ **Tóth, Few 50s**
- $c(n) \leq 1.392\sqrt{n} + 7/4$ **Karloff 89**
- $X \subseteq [0,1]^k$: $\quad c_k(n) = \Theta(n^{1-1/k})$

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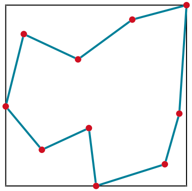

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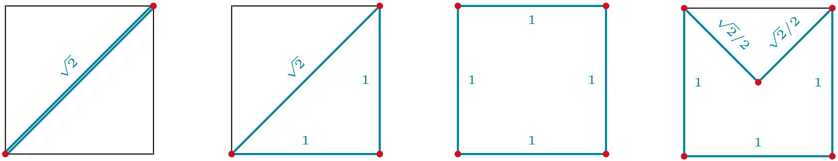

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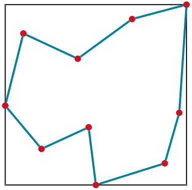

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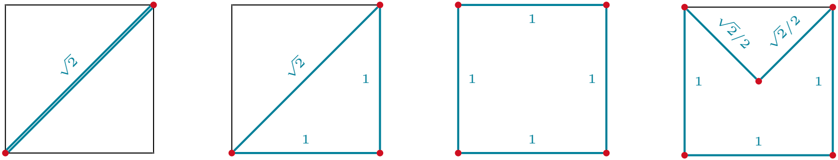

- $c'(n) = 4$ **Newman 82**

does not depend on n!

Newman 82

$\forall$ finite $X \subseteq [0,1]^2$ $\exists$ Ham. cycle $\mathbf{H}$ with $\sum_{\mathbf{H}} |e|^2 \leq 4$

- what if $X \subseteq [0,1]^k$, $k > 2$? **Meir 87**

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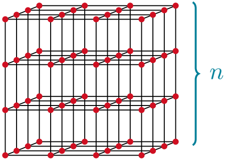

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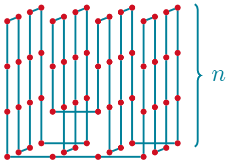

$$\sum |e|^m \approx \frac{n^k}{n^m}$$

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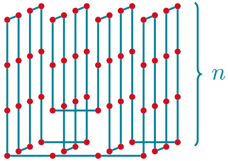
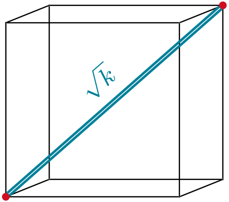


$$\sum |e|^m \approx \frac{n^k}{n^m} \xrightarrow{n\to\infty} \begin{cases} \infty & \text{if } k > m, \\ 0 & \text{if } k < m, \\ \mathbf{1} & \textbf{if } \mathbf{k = m}. \end{cases}$$

c independent of $|X|$ only when $m = k$

? best $\mathbf{c_k}$: $\forall$ finite $X \subseteq [0,1]^k$ $\exists$ Ham. cycle $\mathbf{H}$ with $\sum_{\mathbf{H}} |e|^k \leq \mathbf{c_k}$



◇ $2 \cdot k^{k/2} \leq \mathbf{c_k} \leq \frac{2}{3} \cdot 9^k \cdot k^{k/2}$

Bollobás–Meir 93

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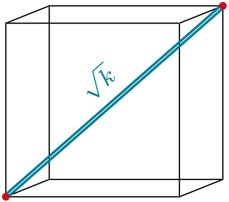

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Bollobás–Meir TSP conjecture

$$\mathbf{c_k} = 2 \cdot k^{k/2} \quad \text{for } k \geq 2$$

- $\mathbf{c_2} = 4$ **Newman 82**

  open for $k > 2$

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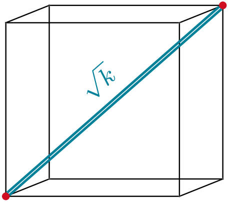
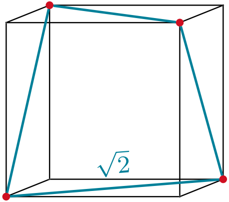




- $2 \cdot k^{k/2} \leq \mathbf{c_k} \leq \frac{2}{3} \cdot 9^k \cdot k^{k/2}$ **Bollobás–Meir 93**

Bollobás–Meir TSP conjecture (updated BCD 24)

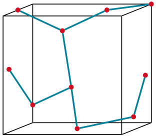
$$\mathbf{c_k} = 2 \cdot k^{k/2} \quad \text{for } k \neq 3, \quad \mathbf{c_3} = 2^{7/2}$$

- $\mathbf{c_2} = 4$ **Newman 82**

  open for $k > 2$

main tools: **cycle approximation** and **ball packing**

- $\mathbf{T}$: min. spanning tree on X



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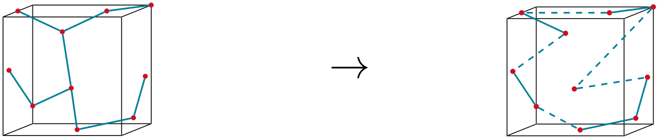

A $\exists$ Ham. cycle $\mathbf{H}$: $\sum_{\mathbf{H}} |e|^k \leq \frac{2}{3} \cdot 3^k \cdot \sum_{\mathbf{T}} |e|^k$ **Sekanina 60s**

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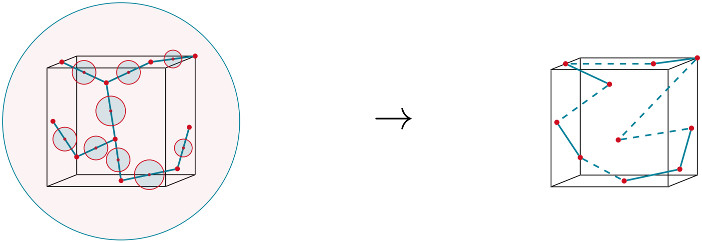

Ⓐ $\exists$ Ham. cycle $\mathbf{H}$: $\sum_{\mathbf{H}} |e|^k \leq \frac{2}{3} \cdot 3^k \cdot \sum_{\mathbf{T}} |e|^k$ **Sekanina 60s**

Ⓑ $\frac{|e|}{4}$-rad. balls on edges of $\mathbf{T}$ are disjoint $\to \sum_{\mathbf{T}} |e|^k \leq 3^k \cdot k^{k/2}$

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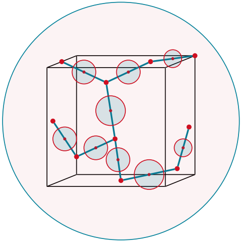

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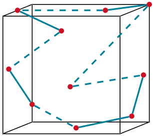

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◇ Ⓐ + Ⓑ $\rightarrow \mathbf{c_k} \leq \frac{2}{3} \cdot 9^k \cdot k^{k/2}$ **Bollobás–Meir 93**

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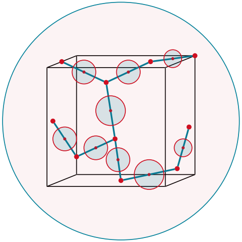

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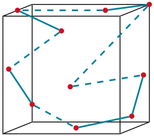

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◇ A + B $\to \mathbf{c_k} \leq \frac{2}{3} \cdot 9^k \cdot k^{k/2}$ **Bollobás–Meir 93**

◇ $\mathbf{c_k} \leq 1.28 \cdot 5.059^k \cdot k^{k/2}$

$\mathbf{c_k} \leq 2.65^k \cdot k^{k/2} \cdot (1 + o_k(1))$ **Balogh–Clemen–Dumitrescu 24, G 25**

used the same combination of packing and approximation

Bollobás–Meir TSP conjecture (updated BCD 24)

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- $\mathbf{c_k} \leq 1.28 \cdot 5.059^k \cdot k^{k/2}$ or $2.65^k \cdot k^{k/2} \cdot (1 + o_k(1))$ **BCD 24, G 25**

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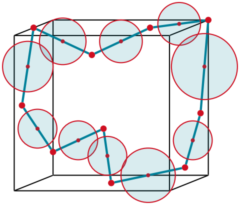
  main new idea: no need to approximate! t-fold packing on $\mathbf{H}$ directly



- $\mathbf{c_k} \le 2\mathrm{e}(k+1) \cdot k^{k/2}$ **G 26**

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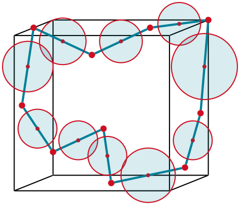
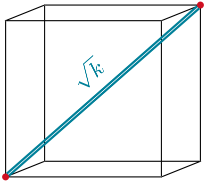


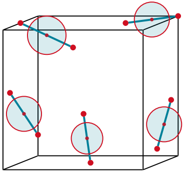


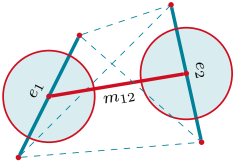
- $\mathbf{c_k} \le 2\mathrm{e}(k+1) \cdot k^{k/2}$ **G 26**
- $\mathbf{c_k} = k^{k/2} \cdot (2 + o_k(1))$ i.e. the conjecture holds asymptotically **G 26**

  $\sum_{\mathbf{H}\setminus\{e_1,e_2\}} |e|^k = o_k(k^{k/2})$, where e_1, e_2 are longest in $\mathbf{H}$

Warm-up

Prove: perfect matching $\mathbf{M}$ with min. $\sum_{\mathbf{M}} |e|^2 \;\Rightarrow\; \frac{|e|}{2\sqrt{2}}$-rad. packing





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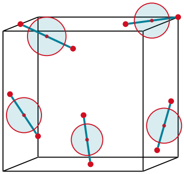

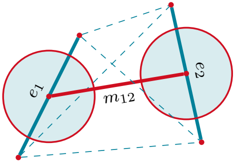

$$\forall a,b,c,d \in \mathbb{R}^k : \left|\frac{a+b}{2} - \frac{c+d}{2}\right|^2 = \frac{|a-c|^2+|b-d|^2+|a-d|^2+|b-c|^2-|a-b|^2-|c-d|^2}{4}$$

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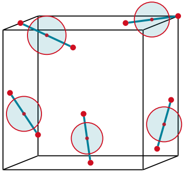

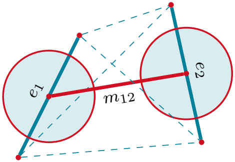

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$\mathbf{M}$:

Warm-up

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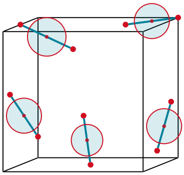

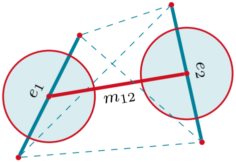

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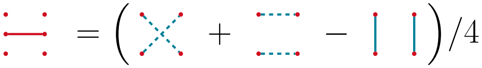
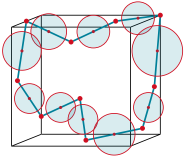


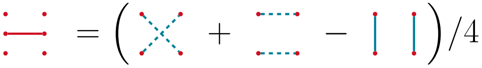
$\mathbf{M}$:

$$|m_{12}|^2 \geq \frac{|e_1|^2+|e_2|^2}{4} \geq \frac{(|e_1|+|e_2|)^2}{8} \quad \Rightarrow \quad |m_{12}| \geq \frac{|e_1|+|e_2|}{2\sqrt{2}}$$

3-fold packing directly on $\mathbf{H}$

Prove: Ham. cycle $\mathbf{H}$ with min. $\sum_{\mathbf{H}} |e|^2 \quad \Rightarrow \quad$ 3-fold $\frac{|e|}{2\sqrt{2}}$-rad. packing





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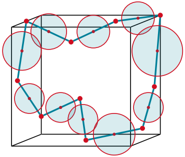

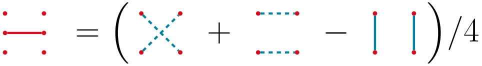

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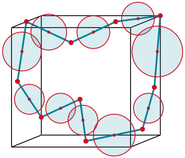

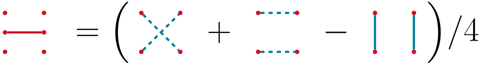

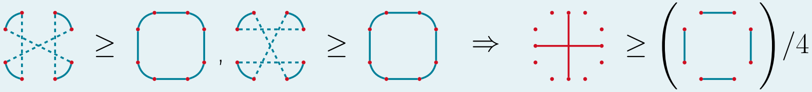

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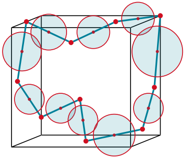
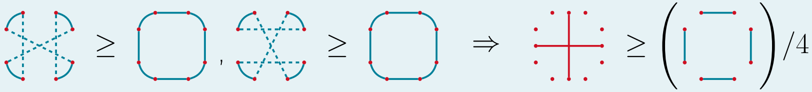
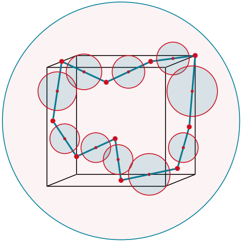




? best $\mathbf{c_k}$: $\forall$ finite $X \subseteq [0,1]^k$ $\exists$ Ham. cycle $\mathbf{H}$ with $\sum_{\mathbf{H}} |e|^k \leq \mathbf{c_k}$

- volume bound on $\frac{|e|}{2\sqrt{2}}$-rad. 3-fold packing



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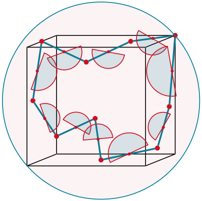
◇ volume bound on $\frac{|e|}{2\sqrt{2}}$-rad. 3-fold half-ball packing $\mathbf{c_k} \leq 6 \cdot (\sqrt{2})^k \cdot k^{k/2}$



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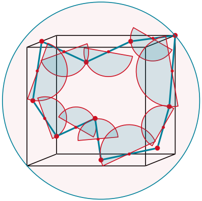

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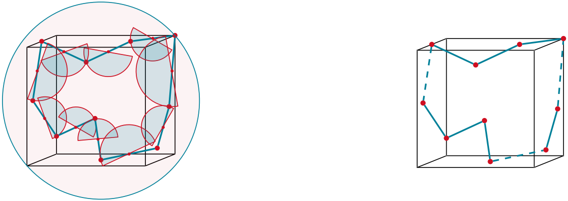

- estimate long edges separately (as **BCD 24** did for spanning trees)

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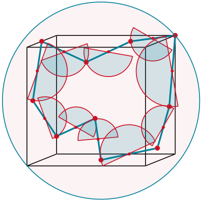

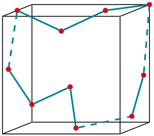

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Open questions

Bollobás–Meir TSP conjecture (updated BCD 24)

$\forall$ finite $X \subseteq [0,1]^k$ $\exists$ Ham. cycle $\mathbf{H}$ on X: $\sum_{\mathbf{H}} |e|^k \leq \mathbf{c_k}$,

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Algorithmic version

Find in polynomial time Ham. cycle $\mathbf{H}$ on $X \subseteq [0,1]^k$: $\sum_{\mathbf{H}} |e|^k \leq \mathbf{c_k^{poly}}$,

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- using **Bender–Chekuri 00** approx. alg.: $\mathbf{c_k^{poly}} \leq 2^{k+2} \cdot k^{k/2} \cdot (1 + o_k(1))$

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A different boundary condition on X (asked in BCD 24)

$\forall$ finite X, $\operatorname{diam} X \leq 1$, $\exists$ Ham. cycle $\mathbf{H}$ on X: $\sum_{\mathbf{H}} |e|^k \leq \mathbf{c'_k}$,
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- $X =$ simplex: $\mathbf{c'_k} \geq k+1$ present methods: $\mathbf{c'_k} \leq 2\mathrm{e} \cdot 2^{k/2} \cdot (k+1)$

Open questions

Thank you!

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